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MODELING OF WATER-DRAIN AND SALT-TRANSFER PROCESSES ON SWAMPED LANDS

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Hydrodynamic and hydraulic models of water drain on swamped lands are proposed, which describe the processes of filtration and surface drain with different degrees of detail and accuracy. Based on the models of salt transfer by interacting filtration and riverbed flows, the issues of modeling the quality of subsoil and surface waters are considered.

Key words: *water transfer, filtration, swamp, river network, modeling.*

Introduction. The water regime of swamped systems is determined by the processes of atmospheric incoming and removal of moisture due to evaporation, surface drain, and filtration in the active layer and in the depth of the peatbog. Removal of subsoil waters from swamps is constrained by the drain properties of the network of rivers and brooks emanating from edge marshes. Part of the surface drain is filtrated in the active peatbog aquifer composed of live plants and their weakly decayed remainders. The highest values of porosity and water permeability and the changes in the level of swamped waters are observed here. Filtration drain from swamped lands, which occurs inside the active layer, passes to peripheral areas, making them superwet and feeding the upper aquifers [1, 2]. An important problem is modeling of mineralization and transfer of pollutants by subsoil waters and evaluation of the influence of the filtration component on the quality of surface waters in aquifers that act as a natural drain system for swamped lands.

We consider models of water drain on swamped lands, which describe the processes of filtration and surface drain.

1. Cross-Sectional Hydrodynamic Model of Water Drain. We consider a two-dimensional hydrodynamic model of saturated-nonsaturated filtration in layered soil [3]. Surface drain is described by the equation of diffusion waves, which allows one to take into account the formation of zones of accumulation and backup modes of swamped water flow. Filtration of subsoil waters is modeled by the Richards equation [4, 5]

$$\theta_t = \operatorname{div}(K\nabla(\psi + y)) + p, \qquad \{(x, y)\colon 0 < x < L,\ H_b < y < H_p(x)\} = \Omega \subset \mathbb{R}^2. \tag{1}$$

Here, $\theta(\psi)$ is the bulk humidity, ψ is the pressure of subsoil waters, $K(\psi)$ is the coefficient of hydraulic conductivity, and H_b and H_p are the coordinates of the confining bed and ground surface. The source function $p(x, y, t) = f - e$ determines moisture absorption by plant roots e and additional infiltration feeding f of surface layers by atmospheric precipitation.

The dependences of the hydraulic conductivity coefficient K and humidity θ on pressure are determined by the formulas

$$K = k_f\Big(\frac{\theta - \theta_1}{m - \theta_1}\Big)^{n_1}, \qquad \theta = \frac{m}{1 + (-\psi/a)^{n_2}}, \qquad \psi < 0,$$

where $K = k_f$ and $\theta = m$ for $\psi \geqslant 0$, $k_f(x, y)$ is the filtration coefficient, θ_1 is the residual humidity, and $m(x, y)$ is the soil porosity. The following values of parameters were used in the calculations: $\theta_1 = 0.05$, $n_1 = 3$, $a = 3$, and $n_2 = 3$. The filtration coefficient and porosity depend on soil lithology and acquire different values in each layer.

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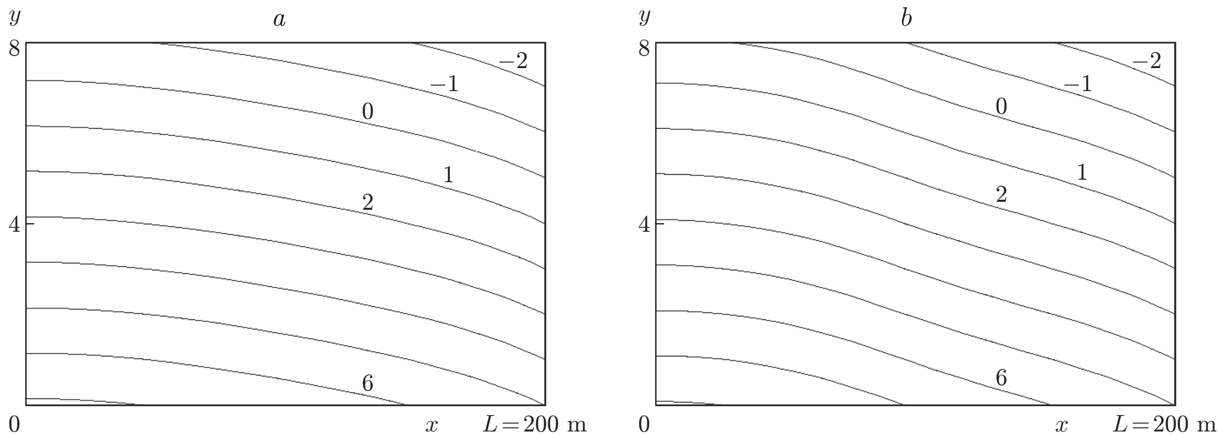

Fig. 1. Pressure isolines of subsoil waters ψ [m] for $t = 62$ (a) and 65 days (b).

For the zone of incomplete saturation on the ground surface ($y = H_p$), we set either the flow rate of water

$$-K \frac{\partial}{\partial n} (\psi + y) = R(x,t) \tag{2}$$

or the pressure of subsoil waters determined by the height of the surface layer of water

$$\psi = z - H_p. \tag{3}$$

The no-slip condition is set on the confining bed $y = H_b$; the side boundaries are subjected either to the condition of symmetry (zero flux) or to dynamic pressure (if the boundary coincides with the aquifer). To close the problem, we have to set the initial distribution of humidity

$$\theta(x, y, 0) = \theta_0(x, y).$$

As the subsoil waters reach the ground surface ($\psi|_{y=H_p} > 0$), the boundary condition (2) is replaced by condition (3) of the first kind, where the dynamic pressure of swamped waters z is determined from the equation of diffusion waves

$$B \frac{\partial z}{\partial t} = \frac{\partial}{\partial x}\Big(\Psi \Big|\frac{\partial z}{\partial x}\Big|^{1/2} \operatorname{sign} \frac{\partial z}{\partial x}\Big) + p(x,t) - k_f \nabla h n\Big|_{y=H_p}, \qquad y = H_p. \tag{4}$$

Here, $\Psi = \gamma(z - H_p)^{5/3}$ is the absolute value of the flow rate of surface drain, h is the filtration pressure, and p is the source function.

The parameters B and γ for $0 < z - H_p < h_k$ are set in the form of linear functions of the thickness of the surface-water layer: $B(x,z) = b + (z - H_p)(1 - b)/h_k$ and $\gamma = \gamma_0 + (z - H_p)(\gamma_1 - \gamma_0)/h_k$ ($\eta_i = 1/\gamma_i$ is the roughness coefficient). If the layer thickness is greater than the critical value ($z - H_p > h_k$), the parameters acquire the values $B = 1$ and $\gamma = \gamma_1$.

For surface drain of swamped waters, the boundary conditions on the left ($x = 0$) and right ($x = L$) boundaries have the form

$$\Psi\Big|\frac{\partial z}{\partial x}\Big|^{1/2} \operatorname{sign} \frac{\partial z}{\partial x} = 0, \quad x = 0, \qquad \frac{\partial z}{\partial x} = \frac{\partial H_p}{\partial x}, \quad x = L. \tag{5}$$

On the basis of this hydrodynamic model, we calculated the cross-sectional problem of coupled filtration and surface drain of swamped waters with allowance for atmospheric precipitation, evaporation, and absorption of moisture by plant roots. The numerical calculations showed that the specific features of the water regime of swamped systems is their sensitivity to small variations of parameters and strong mutual influence of the water-drain components. Intense arrival of moisture from the ground surface leads to rapid saturation of soil, which is caused by vertical flows in a thin aeration zone. Later on, slow unloading into the river network occurs with the help of the filtration flow in the active layer and in the depth of the peatbog.

Figure 1 shows the results for saturated–nonsaturated filtration calculated by model (1)–(5). The modeling domain of length $L = 200$ m consists of uniform soil ($k_f = 1.0$ m/day and $m = 0.3$). On the ground surface

($H_p = 8$ m), we set the flux $R = 0.004$ m/day. At the time $t > 63$ days, subsoil waters reach the ground surface, and there appears a thin layer of surface waters (the upper boundary of the complete saturation zone corresponds to the isoline $\psi = 0$). Drastic changes occur in the subsoil-water regime and pressure distribution in the modeling domain.

Using the hydrodynamic model (1)–(5), one can solve local problems for individual cross sections [3], which allows one to model filtration processes in the active bed and in the complete saturation zone in much detail. Inhomogeneities of the surface relief and peatbog structure, however, as well as aquifers and reservoirs inside the swamp affect the hydrological regime of the entire swamped system. To take into account drain inhomogeneity in the planform, one has to consider three-dimensional hydrodynamic models, which leads to high computational costs. In this case, water transfer in swamped systems can be effectively modeled by planform hydraulic models of coupled drain of subsoil and surface waters.

2. Hydraulic Model of Interaction of Subsoil and Surface Waters. Water-drain processes on swamped lands exert a strong effect on each other, which necessitates their coupled consideration within the framework of a single water-transfer model, in which the common elements for the entire swamped system are the hydrographic network inside the swamp and filtration drain in the active aquifer. Extensive experience has been accumulated for constructing similar models for closed storage basins with some elements of the hydrological cycle included [4, 5]. The most important component of all these models is the submodel of coupled drain of surface and subsoil waters. The flow of subsoil waters is usually modeled by hydraulic equations of unsteady planform unconfined filtration (such as the Boussinesq equations). The vertical infiltration motion of water in the zone of incomplete saturation of soil can be described by the one-dimensional Richards model.

We consider the model of coupled subsoil- and surface-water flows with allowance for vertical migration of moisture in the aeration zone [4]. The model takes into account the main specific feature of water-transfer processes in swamped systems: strong mutual influence of zones of complete and incomplete saturation, vertical inhomogeneity of the peatbog, large contribution of the filtration component to riverbed drain, etc.

The flow of subsoil waters is described by the planform-filtration equation

$$\mu H_t = \operatorname{div}(M \nabla H) + f_1, \qquad x = (x_1, x_2) \in \Omega \subset \mathbb{R}^2, \tag{6}$$

where $H(x,t)$ is the level (pressure) of subsoil waters, μ and M are the coefficients of specific water yield and hydraulic conductivity, and $f_1(x,t)$ is the source function, which takes into account infiltration feeding.

The riverbed drain in the river network Π_i is modeled by a system of one-dimensional equations of diffusion waves, which are approximations of the Saint Venant equations

$$\omega_t = (\Psi |u_s|^{1/2} \operatorname{sign} u_s)_s - [MH_n]_{\Pi_i} + f_2 \qquad (x \in \Pi_i), \tag{7}$$

where $u(s,t)$ is the level of water in the aquifer, $\omega(u,s)$ is the cross-sectional area ($B = \omega_u$ is the width), s is the distance along the aquifer, $\Psi(u,s)$ is the discharge modulus, f_2 is the source function, $[MH_n]_\Pi = (MH_n|_{\Pi+} + MH_n|_{\Pi-})$ is the total filtration inflow of subsoil waters from the right and left banks of the aquifer, and H_n is the external normal derivative.

The level or flux is set at the beginning and end of each aquifer. At points of junction of several aquifers, their levels are identical and the inflow equals the outflow.

The condition of conjugation of surface and subsoil waters is set at the inner boundaries Π_i corresponding to aquifers. For moderate-width aquifers, the levels of subsoil waters at the left and right banks can be assumed to coincide; then, the above-mentioned condition can be written as follows:

$$[MH]_\Pi = MH_n|_{\Pi+} + MH_n|_{\Pi-} = 2\beta(u - H). \tag{8}$$

Vertical migration of subsoil waters in the zone of incomplete saturation is calculated only at the refinement domain D and is described by the one-dimensional Richards equation

$$\theta_t = (K(\psi_y + 1))_y + f_3 \qquad (H < y < H_p,\ x \in D \subset \Omega), \tag{9}$$

where $\theta(\psi)$ is the bulk humidity, ψ is the pressure of subsoil waters (corresponding to the height of the water column), $K(\theta, y)$ is the coefficient of hydraulic conductivity, and y is the vertical coordinate directed upward. In addition, it is necessary to prescribe initial data, the boundary condition (2) on the ground surface, and the condition $\psi|_{y=H} = 0$ on the surface of subsoil waters.

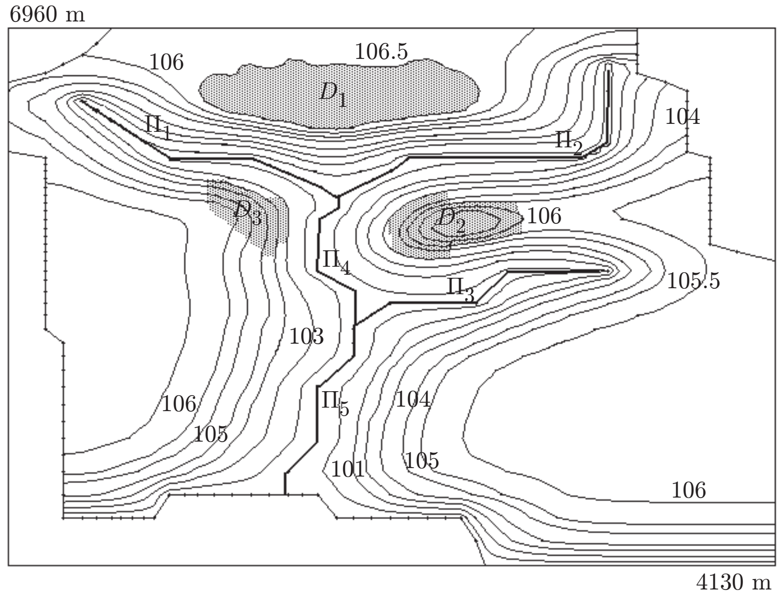

Fig. 2. Isolines of subsoil-water levels.

The interacting system of the zones of complete and incomplete saturation is conjugated on the basis of the condition of the balance of interstitial moisture transferred by vertical flows in the aeration zone and by horizontal subsoil-water flows [4]. In this case, instead of Eq. (6), the refinement is described by the equation

$$0 = \operatorname{div}(\bar{M}\nabla H) - K(\psi_y + 1)\Big|_{H_p} + f, \qquad x = (x_1, x_2) \in \Omega. \tag{6'}$$

The coefficient of hydraulic conductivity is determined as the sum of hydraulic conductivities in the zones of complete and incomplete saturation

$$\bar{M}(x,t) = \int\limits_{H_b(x)}^{H(x,t)} k_f(x,y)\,dy + \int\limits_{H(x,t)}^{H_p(x)} K(\psi(x,y,t))\,dy.$$

The additional second term takes into account the horizontal component of filtration drain in the aeration zone, where the condition $\nabla H \approx \nabla_x \psi$ is assumed to be satisfied.

Atmospheric precipitation, evaporation from the ground surface, and absorption of moisture by plant roots are taken into account by the stream function in the boundary condition for the aeration zone and by the source functions in the corresponding equations.

The results of numerical simulations by the approximate hydraulic model (6′), (9) are compared in [6] with the results calculated by the hydrodynamic model based on the Richards equation (1) by an example of cross-sectional model problems.

We give an example of a river-network fragment with periodically changing data (the period equals 365 days), which was calculated by model (6)–(9), (6′). The river network consists of five aquifers Π_i (i = 1, 2, 3, 4, and 5), which have common points of junction (Fig. 2). The aquifer widths are 1.3, 1.2, 1.6, 1.7, and 1.6 m, the parameter β in condition (8) of conjugation of subsoil and surface waters equals 9.0, 10.0, 12.0, 10.5, and 13.2 m/day, respectively, $\gamma_1 = \gamma_2 = 1.4 \cdot 10^6$, and $\gamma_3 = \gamma_4 = \gamma_5 = 2.0 \cdot 10^6$.

The no-slip condition was set at the upper and side boundaries of the simulation domain, corresponding to watersheds, and a condition of the first kind was prescribed at the lower boundary. The subsoil-water flow was modeled by the equation of unconfined filtration (6) with the coefficient of filtration of the main soil thickness equal to 0.4 m/day. Near the ground surface, there is a soil layer 2 m thick with a filtration coefficient of 8 m/day, which simulates the active bed. In the entire domain, the specific water yield is $\mu = 0.1$, and the infiltration f_1 is piecewise-constant in time: $f_1 = 0.0001$ m/day for $0 < t < 100$ days, $f_1 = 0.0003$ m/day for 100 days $< t <$ 200 days, and $f_1 = 0.0002$ m/day for 200 days $< t <$ 365 days. The heights of the ground surface and confining bed are $H_p = 107$ m and $H_b = 85$ m, respectively.

The domain consists of three sections with intense infiltration feeding (R) of subsoil waters, where the problem of moisture migration in the zone of incomplete saturation was solved with the use of Eqs. (6′), (9) (refinement domains D_1, D_2, and D_3; $R = 0.0001$ m/day for $0 < t < 100$ days, $R = 0.0006$ m/day for 100 days $< t <$ 200 days, and $R = 0.0002$ m/day for 200 days $< t <$ 365 days).

The numerical calculations were performed within a two-year interval with the initial level of subsoil waters $H_0 = 105$ m. Figure 2 shows the isolines of the levels of subsoil waters for the time $t = 565$ days with a 1-m step between the isolines (in addition, two intermediate isolines corresponding to the levels of 105.5 and 106.5 m are given). For this time, the depths of aquifers at the junction points (Π_1, Π_2) and (Π_3, Π_4) are 0.21 and 0.18 m, respectively.

This example is the problem with strong interaction of subsoil and surface waters, despite a comparatively small size of the storage basin (4130 × 6960 m). The qualitative and quantitative characteristics of subsoil- and surface-water flows are determined by the intensity of water transfer between various components of water drain. Within the entire computational domain, the river network drains subsoil waters, and the drain is formed only due to filtration inflow. In refinement domains D_1 and D_2, subsoil waters reach the ground surface ($H > H_p$) at some time instants with high values of infiltration feeding.

The above-considered hydraulic model is based on conjugation of the two-dimensional filtration equation and one-dimensional equations in the zone of incomplete saturation, which significantly decreases the computational costs in calculating water-transfer processes. Nevertheless, if the aeration zone is composed of well permeable soils and infiltration feeding (R) changes little in time, the pressure distribution of subsoil waters is rather steady:

$$K(\psi_y + 1) = R, \qquad H < y < H_p. \tag{10}$$

In this case, the changes in humidity in the aeration zone can be described by a simpler model with the use of the balance relation. For the volume of subsoil waters $V = \int\limits_{H_b}^{H_p} \theta(\psi)\, dy$ in the soil layer from the confining bed to the ground surface, we can consider the differential equation

$$\frac{\partial V}{\partial t} = \operatorname{div}(\bar{M}\nabla H) + R, \qquad x = (x_1, x_2) \in \Omega. \tag{11}$$

The hydraulic conductivity $\bar{M}$, as in Eq. (6′), is the total conductivity of the zones of complete and incomplete saturation. In the case of a constant infiltration feeding $R = \text{const}$, the function $\bar{m} = dV/dH$ describes the specific water yield.

Some results on the cross-sectional saturated-nonsaturated filtration, which were calculated by the hydrodynamic model (1)–(5) and hydraulic model (10), (11), are given below.

In variants 1 and 2, the soil is homogeneous: $k_f = 1.0$ m/day, $m = 0.6$ ($0 < y < 8$ m); $R = 0.0025$ m/day, and the lengths L of the regions in the x direction are different and equal to 200 and 100 m, respectively. In variant 3, a two-layer structure of soil is considered: $k_f = 0.2$ m/day for $m = 0.6$ ($0 < y < 4$ m) and $k_f = 1.0$ m/day for $m = 0.7$ (4 m $< z <$ 8 m); $R = 0.004$ m/day and $L = 200$ m.

The pressure isolines in the zone of incomplete saturation (a) calculated by the hydrodynamic model and the levels of subsoil waters (b) obtained by solving the problem in the hydraulic approximation ($t = 125$ days) are plotted in Fig. 3 for variant 1. In addition, Fig. 3b shows the water yield and hydraulic conductivity.

The calculated values of the levels of subsoil waters at the point $x = 0$, obtained by the two models mentioned above, are listed in Table 1. As the subsoil waters approach the ground surface, the rate of change in their position increases, which reduces the accuracy of determining the level of subsoil waters in the hydraulic model.

3. Modeling of the Quality of Subsoil and Surface Waters. Models of salt transfer by interacting flows of subsoil and surface waters are based on equations of convective diffusion and take into account the salt exchange between water-drain components [7]. In the planform hydraulic model (6)–(9), (6′), salt concentrations are averaged over the thickness and cross section of aquifers.

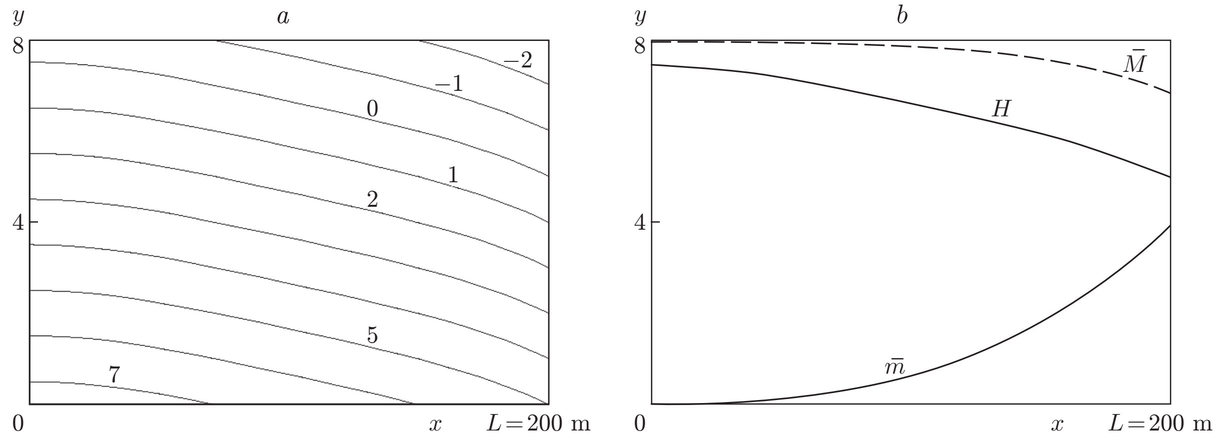

Fig. 3. Pressure isolines of subsoil waters (a) and level of subsoil waters (b).

TABLE 1

Calculated Values of Subsoil-Water Levels

Variant 1			Variant 2			Variant 3		
t, days	$H(0,t)$, m		t, days	$H(0,t)$, m		t, days	$H(0,t)$, m	
	Hydrodynamic model	Hydraulic model		Hydrodynamic model	Hydraulic model		Hydrodynamic model	Hydraulic model
50	5.476	5.471	50	5.459	5.457	50	5.702	5.697
100	6.239	6.238	100	5.925	5.930	65	6.025	6.025
120	6.957	6.965	150	6.302	6.307	75	6.338	6.345
125	7.514	7.552	300	6.660	6.656	85	7.031	7.110

Salt transfer by the filtration flow in the aquifer is described by the two-dimensional equation

$$(mdC)_t = \operatorname{div}(D\nabla C - vC) - \Phi(C,N) + fC_f^*, \tag{12}$$

where $m = m_0 + \mu(H - H_p)/d$ $(d = H_p - H_b)$, $v = -M\nabla H$, C and N are the salt concentrations in the solution and in the solid phase, respectively, H_p and H_b are the heights of the top and bottom of the aquifer, respectively, $C_k = \text{const}$, and $D = dD_0 + \lambda|v|$ is the diffusion coefficient.

Equation (12) is supplemented by the boundary conditions

$$(D\nabla C - vC)n\Big|_{\partial\Omega} = -vnC_g^*\Big|_{\partial\Omega}, \qquad Q_f \equiv (D\nabla C - vC)n\Big|_{\Pi} = -vnC^*\Big|_{\Pi}.$$

The values of C^* are determined by the filtration-flow direction

$$C_g^*\Big|_{\partial\Omega} = \{C,\ q < 0;\ C_g,\ q \geqslant 0\},$$

$$C^*\Big|_{\Pi} = \{C,\ q < 0;\ C_1,\ q \geqslant 0\}, \qquad C_f^* = \{C,\ f < 0;\ C_f,\ f \geqslant 0\},$$

where C_g and C_f are prescribed functions and $q = -vn$.

For a nonconservative admixture, the process of salt deposition on the soil skeleton is determined by the ordinary differential equation

$$(dN)_t = \Phi(C,N) = pN_0(C - C_k).$$

Transfer of admixtures by riverbed drain is modeled by the system of one-dimensional equations

$$(\omega C_1)_t = (D_1(C_1)_s - v_1C_1)_s + [vnC^*]_\Pi + f_1C_1^*, \quad (x,y) \in \Pi, \qquad v_1 = -\Psi|z_s|^{1/2}\operatorname{sign} z_s, \tag{13}$$

where C_1 is the salt concentration in the aquifer and $D_1 = \omega D_K + \lambda_1|v_1|$.

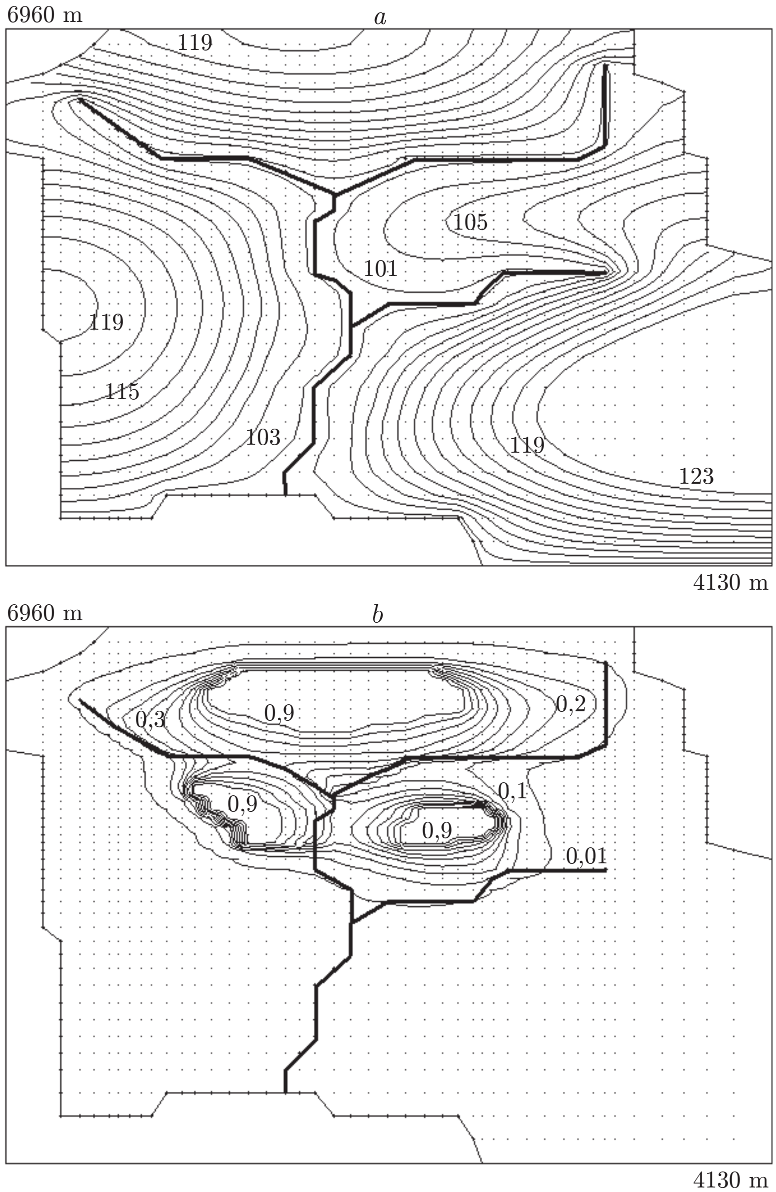

Fig. 4. Isolines of the level of subsoil waters (a) and admixture concentration (b).

At the beginning and end of aquifers (at the points P), we set the boundary conditions of the form

$$(D_1(C_1)_s - v_1 C_1)\Big|_P = -v_1 C_P^*\Big|_P.$$

To close the salt-transfer problem, we have to set the initial values of salt concentration for all water-drain components.

As an example, we consider the model problem (6)–(8), (12), (13) of transfer of a nonconservative admixture by interacting flows of subsoil and surface waters. The simulation domain and the aquifer parameters correspond to the example considered in Sec. 2 (see Fig. 2). The confined flow of subsoil waters is modeled by the linearized equation of planform filtration (6) with a porosity coefficient $m = \mu = 0.2$ and hydraulic conductivity coefficient $M = 8$ m^2/day increasing to $M = 20$ m^2/day near aquifers. In the domains D_i ($i = 1, 2, 3$), the flow in the incomplete saturation zone was not calculated. In this present example, the materials composing the aquifer in these domains (D_i) contain admixtures soluble in subsoil waters ($N_0 = 9.8$). In the remaining part of the simulation domain, there are no admixtures in the aquifer ($N_0 = 0$). Admixture transfer by the filtration flow was modeled with salt-transfer parameters $d = 10$ m, $D_0 = 0.1$ m^2/day, and $\lambda = 10$ m and the desorption kinetics parameter $p = 2.5 \cdot 10^{-4}$ m/day. The parameters of salt transfer by riverbed drain were $D_K = 20$ m^2/day and $\lambda_1 = 100$ m. Infiltration f was described by a piecewise-smooth function in time: $f_1 = 0.00005$ m/day for $0 < t < 100$ days, $f_1 = 0.0003$ m/day for 100 days $< t <$ 200 days, and $f_1 = 0.0002$ m/day for 200 days $< t <$ 365 days.

The numerical calculations were performed until a periodic regime was reached. Figure 4 shows the isolines of subsoil-water levels (with a 2-m step between the isolines) and admixture concentrations (with a 0.1 step and an additional isoline for the concentration equal to 0.01) for the time corresponding to the end of the annual period (365 days). For this time, the depths of aquifers at the junction points (Π_1, Π_2) and (Π_3, Π_4) are 0.24 and 0.21 m, respectively. Pollution of surface waters in the river network is determined by inflow of an admixture with a filtration component to aquifers. Polluted subsoil waters enter the aquifers Π_1, Π_2, and Π_3 in the upper part of the river network, whereas the surface waters at the river mouth are diluted by filtration inflow with an insignificant concentration of the admixture. At the end of the annual period, the following concentrations are reached at the initial and final points of the aquifers, respectively: 0.020 and 0.246 (aquifers 1), 0.008 and 0.151 (2), 0.197 and 0.199 (3), 0 and 0.019 (4), and 0.149 and 0.116 (5). The concentration in aquifers changes only slightly during the year, which is caused by a comparatively constant content of the admixture in subsoil waters.

Conclusions. Models of water systems (water reservoirs, aquifers, filtration of subsoil waters, etc.) composing the overall model of water transfer are not identical in terms of their complexity and have different dimensions. It is of particular importance that the characteristic time scales of transitional processes in surface and subsoil waters differ by orders of magnitude, which is relevant for numerical simulation of coupled water-drain processes. Additional requirements to computational algorithms used are imposed in considering problems of salt transfer by interacting flows. Difference schemes for individual components of mass transfer should be conservative, and the conditions of conjugation should take into account the specific features of water and salt balance of the entire hydrological cycle.

The hydrodynamic and hydraulic models considered allow one to model water-drain processes with different degrees of refinement and accuracy. In considering particular problems on swamped lands, the choice of an adequate model requires taking into account the complicated character of water transfer, spatial and time scales of the hydrological processes considered, and the degree of completeness of available information. Mathematical simulation of the quality of subsoil and surface waters makes it possible to evaluate the environmental situation for real water objects and model measures compensating for undesirable man-induced actions.

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